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Dual Superconductivity in Restricted Chromo-Dynamics (RCD)

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Abstract Characterizing the dyonically condensed vacuum by the presence of two massive modes (one determining how fast the perturbative vacuum around a colour source reaches the condensation and the other giving the penetration length of colored flux), it has been shown that due to the dynamical breaking of magnetic symmetry the vacuum of RCD acquires the properties similar to those of relativistic superconductor. Originally present global $SU(2)$ symmetry in RCD has been broken to $U(1)$ reducing the four dimensional action to two dimensional one by using an Ansatz which incorporates a non-trivial coordinate dependent phase between the components of $SU(2)$ doublet. Analyzing the behaviour of dyons around RCD string, the solutions of classical field equations have been obtained and it has been shown that magnetic constituent of dyonic current is zero at centre of the string and also at the points far away from the string. The conditions for this current to be maximum at a transverse distance from the string have also been obtained.

Keywords Restricted chromodynamics · Penetration length · Dyons · Confinement · Condensation · Meissner effect

1 Introduction

In the process of current understanding of superconductivity at high T_c, one conceives the notion of its hopeful analogy with quantum chromo-dynamics (QCD). The essential clues

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for gauge symmetry breaking emerged from the crucial theoretical frame work of BCS theory [1] of superconductivity. Other silent features of superconductivity viz. the Meissner effect and the flux quantization provided the vivid models for actual confinement mechanism. In this connection Nambu [2, 3] and others [4–6] suggested that the colour confinement could occur in QCD in a way similar to magnetic flux confinement in superconductors. Mandelstam [7–9] elaborated it by expounding that the colour confinement properties may result from the condensation of magnetic monopoles in QCD vacuum. In a series of papers Ezawa and Iwazaki [10–13] made an attempt to analyse a mechanism of quark confinement by demonstrating that the Yang-Mills vacuum is a magnetic superconductor and such a superconducting state is considered to be a condensed state of monopoles or magnetic vortices. The condensation of monopoles incorporates the state of magnetic superconductivity [14] and the notion of chromo-magnetic superconductor [15] where the Meissner effect confining magnetic field in ordinary superconductivity would be replaced by dual Meissner effect which would confine the colour electric field. It leads to a correspondence between quantum chromo-dynamic situation and chromo-magnetic superconductor, where the Abelian electric field is squeezed by solenoidal monopole current [16, 17] and the colour confinement takes place due to dual Meissner effect caused by monopole condensation.

Using this idea of confinement of electric flux due to condensation of magnetic monopoles, a dual gauge theory called restricted chromo dynamics (RCD) has been constructed out of QCD in $SU(2)$ theory [18–21]. This dual gauge theory incorporates a dynamical dyonic condensation [22, 23] and exhibits the desired dual dynamics that guarantees the confinement of dyonic quark through generalized Meissner effect. This RCD has been extracted from QCD by imposing an additional internal symmetry named magnetic symmetry [18, 24] which reduces the dynamical degrees of freedom. Attempts have been made [25] to establish an analogy between superconductivity and the dynamical breaking of magnetic symmetry, which incorporates the confinement phase in RCD vacuum.

In the present paper the formulation of RCD has been extended in the light of the concept of chromo-dyonic superconductor and it has been shown that in the confinement phase the dyonic condensations of vacuum gives rise to the complex screening current which confines both the chromo-electric and chromo-magnetic fluxes through the mechanism of generalized Meissner effect (the usual one and its dual). Characterizing the dyonically condensed vacuum by the presence of two massive modes (one determining how fast the perturbative vacuum around a colour source reaches the condensation and the other giving the penetration length of the coloured flux), it has been shown that due to the dynamical breaking of magnetic symmetry the vacuum acquires the properties similar to those of relativistic super-conductor where the quantum fields generate non-zero expectation values and induce screening currents. It has also been shown that the generalized charge space parameter associated with dyons has the remarkable ability to squeeze the colour fluxes and to improve the confining properties of RCD vacuum. Originally present global $SU(2)$ symmetry in RCD has also been broken to $U(1)$ reducing the four dimensional action to the two dimensional one by using an Ansatz [26, 27] for the four components of vector field and two complex components of Higgs field, which incorporates a non-trivial coordinate dependent phase between the components of $SU(2)$ doublet, Taking special cases of this Ansatz the untwisted static solutions and twisted stationary solutions for RCD strings have been obtained. Analyzing the behaviour of dyons around the RCD string with a quark and an anti-quark at its ends, the static solutions of classical field equations have been obtained removing the mistakes in the recent relations of Chernodub et al. [28] and it has been shown that the magnetic constituent of the dyonic current is zero at centre of the string and also at points far away from the string. The conditions for this current to be maximum at a transverse distance from the string have been obtained.

2 Magnetic Symmetry and Restricted Chromo Dynamics (RCD)

Mathematical foundation of RCD [18, 21] is based on the fact that a non-Abelian gauge theory permits some additional internal symmetry i.e. the magnetic symmetry. Let us briefly review the RCD in the $(4+n)$ dimensional metric manifold P (four-dimensional space-time manifold M and n-dimensional internal group G) with metric g_{AB} $(A, B = 1, 2, \ldots, 4+n)$, where the gauge symmetry can be viewed as n-dimensional isometry [29, 30] which allows us to view P as a principal fibre bundle $P(M, G)$ with $M = P/G$ as the base manifold and G as the structure group. Keeping in view the fact [18] that the restricted theory RCD may be extracted from full QCD by imposing an extra internal symmetry, let us now restrict the dynamical degrees of freedom of the theory (keeping full gauge degrees of freedom intact) by imposing an extra magnetic symmetry which ultimately forces the generalized non-Abelian gauge potential $\vec{V}_\mu$ to satisfy a strong constraint given by

$$D_\mu \hat{m} = \partial_\mu \hat{m} + i|q|\vec{V}_\mu x \hat{m} = 0 \tag{2.1}$$

where D_μ is covariant derivative for the gauge group, $\mu = 0, 1, 2, 3$, $q = (e - ig)$ is the generalized dyonic charge with e and g as electric and magnetic constituents, and the generalized four-potential $\vec{V}_\mu$ is given as

$$\vec{V}_\mu = \vec{A}_\mu - i\vec{B}_\mu \tag{2.2}$$

where A_μ and B_μ are electric and magnetic four-potentials respectively. The cross product in (2.1) is taken in internal group space and $\hat{m}$ characterizes the additional Killing symmetry-(magnetic symmetry) which commutes with the gauge symmetry itself and is normalized to unity i.e.

$$\hat{m}^2 = 1. \tag{2.3}$$

It constitutes an adjoint representation of G, whose Little group is assumed to be Cartan sub-group [18] at each space-time point. Mathematically, this means that a connection on $P(M, G)$ admits a left isometry of H, which formally forms a subgroup of G but commuts with G (the right isometry). This magnetic symmetry restricts the connection (i.e. the space for potential) to those whose holonomy bundle becomes a reduced bundle $P(M, H)$.

Choosing $G = SU(2)$ and $H = U(1)$, the gauge covariant condition (2.1) gives the following form of the generalized restricted potential,

$$\vec{V}_\mu = -iV_\mu^* \hat{m} + (i/|q|)\hat{m} x \partial_\mu \hat{m} \tag{2.4}$$

such that $\hat{m}\vec{V}_\mu = -iV_\mu^*$ is the unrestricted Abelian component of the restricted potential $\vec{V}_\mu$ while the remaining part is completely determined by magnetic symmetry.

The unrestricted part of the gauge potential describes the dyonic flux of color isocharges and the restricted part describes the flux of topological charges of the symmetry group G. The imposed magnetic symmetry, revealing the global topological structure of gauge symmetry, enables us to conceive the gauge theory of non-trivial fibre bundle $P(M, H)$ with only those fields which are defined on global sections where color direction would be chosen by selecting color electric potential of Cartan's sub-group which helps to circumvent the disturbing Schlieder's theorem [31] in defining a meaningful color charge in non-Abelian gauge theory.

The generalized field strength of the gauge field of RCD that describes non-Abelian dyons may be obtained as follows [15, 22, 23];

$$\begin{aligned}\underline{\vec{G}}_{\mu\upsilon} &= \vec{G}_{\mu\upsilon} + (i/|q|)[\vec{V}_\mu \times \vec{V}_\upsilon],\\ &= (-iF_{\mu\upsilon} + H_{\mu\upsilon})\hat{m}\end{aligned} \tag{2.5}$$

where

$$\vec{G}_{\mu\upsilon} = \vec{V}_{\upsilon,\mu} - \vec{V}_{\mu,\upsilon},$$
$$F_{\mu\upsilon} = V^*_{\upsilon,\mu} - V^*_{\mu,\upsilon},$$

and

$$H_{\mu\upsilon} = (i/|q|)\hat{m}.[\partial_\mu \hat{m} \times \partial_\upsilon \hat{m}]. \tag{2.6}$$

Identifying $F_{\mu\upsilon}$ and $H_{\mu\upsilon}$ as the generalized electric and magnetic field strengths respectively, the striking duality between the generalized electric and magnetic fields is obviously manifested in the theory. These field strengths satisfy the following dual symmetric field equations in magnetic gauge

$$F_{\mu\upsilon,\upsilon} = j_\mu \quad \text{and} \quad H_{\mu\upsilon,\upsilon} = -k_\mu \tag{2.6a}$$

where j_μ and k_μ are respectively the electric and magnetic four-current densities constituting the generalized dyonic four-current density

$$J_\mu = j_\mu - ik_\mu. \tag{2.6b}$$

In order to demonstrate the topological structure, let us introduce magnetic gauge by aligning $\hat{m}$ along a space-time independent direction (say $\hat{\varepsilon}_3$ in isospin space) by imposing a gauge transformation U such that

$$\hat{m} \xrightarrow{U} \hat{\varepsilon}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.7}$$

and the potential and field strength transform as

$$\vec{V}_\mu \to \vec{V}'_\mu = (-iV^*_\mu + W_\mu)\hat{\varepsilon}_3$$

and

$$\underline{\vec{G}}_{\mu\upsilon} = \underline{\vec{G}'}_{\mu\upsilon} = (-iF_{\mu\upsilon} + H_{\mu\upsilon})\hat{\varepsilon}_3 \tag{2.8}$$

with

$$H_{\mu\upsilon} = W_{\upsilon,\mu} - W_{\mu,\upsilon} \tag{2.9}$$

where W_μ may be identified as the potential of topological dyons in magnetic symmetry which is entirely fixed by $\hat{m}$ upto Abelian gauge degrees of freedom. Thus in the magnetic gauge, the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_μ by removing all non-essential gauge degrees of freedom and hence the topological structure of the theory may be brought into dynamics explicitly. It assures a non-trivial dual structure of the theory of dyons in magnetic gauge where dyons appear as point like Abelian ones.

In this theory the gauge fields are expressible in terms of purely time like non-singular physical potentials V_μ^* and W_μ. Following Mandelstam [7] and t' Hooft [4], let us introduce a complex scalar field ϕ (Higgs field) to eliminate the point like behaviour and to incorporate the extended structure of dyons. Then in the absence of quarks or any colored object, the RCD Lagrangian in magnetic gauge may be written as

$$L = \left(\frac{1}{4}\right) H_{\mu\upsilon} H^{\mu\upsilon} + \left(\frac{1}{2}\right) |D_\mu \phi|^2 - V(\phi^* \phi) \tag{2.10}$$

where $D_\mu \phi = (\partial_\mu + i|q|W_\mu)\phi$ and $V(\phi^*\phi)$ is the effective potential introduced to induce the dynamical breakdown of the magnetic symmetry.

3 Dyonic Condensation and Super-Conductivity in Restricted Chromodynamics

The Lagrangian equation (2.10) of RCD in magnetic gauge in the absence of quark or any colored object, looks like Ginsburg-Landau Lagrangian for the theory of superconductivity if we identify the dyonic field as an order parameter and the generalized potential W_μ as the electric potential. The dynamical breaking of the magnetic symmetry, due to the effective potential $V(\phi * \phi)$, induces the dyonic condensation of the vacuum. This gives rise to the dyonic super current, the real part of which (electric constituent) screens the electric flux which confines the magnetic color charge (through usual Meissner effect) and the imaginary part (i.e. magnetic constituent) of this super-current screens the magnetic flux that confines the electric color iso-charges (due to dual Meissner effect). In other words, the dual Meissner effect expels the electric field between static colored charges in to a narrow flux tube, giving rise to a linearly rising potential and to confinement [32].

Lagrangian (2.10) has been obtained from the standard $SU(2)$ Lagrangian and hence the desired dynamical breaking of magnetic symmetry is obtained by fixing the following form of the effective potential;

$$V(\phi^*\phi) = -\eta(|\phi|^2 - \upsilon^2)^2 \tag{3.1}$$

where η is compling constant of Higgs field and υ is its vacuum expectation value i.e.

$$\upsilon = \langle \phi \rangle_0. \tag{3.2}$$

In Prasad-Sommerfeld limit [33],

$$V(\phi) = 0$$

but $\upsilon \neq 0$.

In this limit, the dyons have lowest possible energy for given electric and magnetic charges e and g respectively. In this Abelian Higgs model of RCD in magnetic symmetry, W_μ defined by (2.9) is the dual gauge field with the mass of dual gauge boson given by

$$M_D = |q|\upsilon, \tag{3.3}$$

and ϕ is the dyonic field with charge q and mass

$$M_\phi = \sqrt{(8\eta)}\upsilon. \tag{3.4}$$

In the confinement phase of RCD the dyons are condensed under the condition (3.2). With these two mass scales the coherence length ε and the penetration length λ are given by

$$\begin{aligned} \varepsilon &= 1/M_\phi = 1/[\sqrt{(8\eta)}v] \quad \text{and} \\ \lambda &= 1/M_D = 1/(|q|v). \end{aligned} \tag{3.5}$$

The region in phase diagram space, where $\varepsilon = \lambda$, constitutes the border between type-I and type-II super-conductors. The super-conductivity provides vivid model for the actual confinement mechanism where the color confinement is due to the generalized Meissner effect caused by dyonic condensation. The dual superconductivity model proposed recently by D'Alessandro et al. [32] places the Yang-Mills vacuum close to the border between type-I and type-II superconductors and marginally on the type-II side.

The Lagrangian (2.10), with effective potential given by (3.1), describes the extended Abelian Higgs model which belongs to the interesting class of models in which both the global $SU(2)$ and local $U(1)$ symmetries are broken simultaneously with a minimal number of scalar fields. In other words it is a semi local model [26] where due to the form of potential, given by (3.1), the scalar field has a vacuum expectations value v and the $U(1)$ local gauge symmetry is simultaneously broken.

Let us scale the fields and coordinates in the following manner [26],

$$\begin{aligned} W_\mu &\to vW_\mu, \qquad \phi \to v\phi \quad \text{and} \\ x^\mu &\to x^\mu/|q|v \end{aligned} \tag{3.6}$$

and the various terms of the Lagrangian (2.10) are then rescaled as

$$\begin{aligned} H_{\mu\upsilon} &\to |q|v^2 H_{\mu\upsilon}, \\ V(\phi^\dagger\phi) &\to -\eta v^4(|\phi|^2-1)^2 \end{aligned} \tag{3.7}$$

and

$$D_\mu\phi \to v^2|q|[\partial_\mu + iW_\mu]\phi = v^2|q|D_\mu\phi \quad \text{where } D_\mu = \partial_\mu + iW_\mu.$$

Then the Lagrangian (2.10) becomes

$$L = |q|^2 v^4\left[\frac{1}{4}H_{\mu\upsilon} + \frac{1}{2}D_\mu\phi D^\mu\phi + \beta(|\phi|^2-1)^2\right] \tag{3.8}$$

where $\beta = \eta/|q|^2$, and the action

$$A = \int Ld^4x$$

is rescaled to

$$A \to A = 1/|q|^2\int\left[\frac{1}{4}H_{\mu\upsilon}H^{\mu\upsilon} + \frac{1}{2}D_\mu\phi D^\mu\phi + \beta(|\phi|^2-1)^2\right]. \tag{3.9}$$

This Lagrangian yields the following field equations

$$\partial_\upsilon H^{\mu\upsilon} = i\phi^\dagger D_\mu\phi = j_\mu^0 \quad \text{and} \tag{3.10}$$

$$D_\mu^2\phi = 4\beta[|\phi|^2-1]\phi \tag{3.11}$$

where

$$j_\mu^0 = i\phi^\dagger[\partial_\mu + iW_\mu]\phi$$

and

$$\begin{aligned} j_\mu^a &= i[\phi^\dagger \tau^a D_\mu \phi] \\ &= i[\phi^\dagger \tau^a (\partial_\mu + iW_\mu)\phi], \end{aligned}$$

with $a = 1, 2, 3$, and i^a Pauli matrices constitute the conserved Notherian current.

Using relation (2.9), we may write (3.10) as

$$\Box W_\mu - \partial^\upsilon \partial_\mu W_\upsilon = i\phi^\dagger \phi[\partial_\mu \phi/\phi + iW_\mu], \tag{3.12}$$

which reduces to the following form in the Lorentz gauge

$$\Box W_\mu = i\phi^+ \phi[\partial_\mu \phi/\phi + iW_\mu],$$

which further reduces in to the following simple form for the small variation in ϕ;

$$\Box W_\mu + |\phi|^2 W_\mu = 0, \tag{3.13}$$

which is a massive vector type equation where the equivalent mass of the vector particle state (condensed mode) may be identified as

$$M = |\phi|$$

with its vacuum expectation value

$$\langle M \rangle = v,$$

which gives

$$M_D = |q|\langle M \rangle = v|q| = 1/\lambda, \tag{3.14}$$

where λ is penetration length. Thus the penetration length directly follows from the field equations (3.10), obtained from the Lagrangian (3.8) of the extended Abelian Higgs model in restricted chromodynamics. Comparing this penetration length λ with that of relativistic superconducting model i.e.

$$M_S = \sqrt{2}e|\phi| = \sqrt{2}ev = 1/\lambda_S,$$

where e is the electric charge of dyons, we get

$$M_D/M_S = \lambda_S/\lambda = |q|/(e\sqrt{2}) = \left(\frac{1}{\sqrt{2}}\right)[(e^2 + g^2)^{1/2}]/e. \tag{3.15}$$

In the representation of generalized charge of dyon in a two dimensional complex space [34], we have

$$g/e = -\tan\theta \tag{3.16}$$

where θ is rotation parameter of the generalized charge space. Then (3.15) becomes

$$\lambda/\lambda_S = \sqrt{2}\cos\theta \tag{3.17}$$

showing that for the rotation parameter $\theta \leq \pi/4$, we have

$$\lambda \geq \lambda_S \quad \text{and} \quad M_D \leq M_S. \tag{3.18}$$

On the other hand, for larger rotation in generalized charge space with $\theta > \pi/4$, we have

$$\lambda < \lambda_S \quad \text{and} \quad M_D > M_S. \tag{3.19}$$

Thus the optimum RCD generalized charge orientation is governed by rotation parameter value $\theta = \pi/4$.

It shows that with a suitable choice of the generalized charge space parameter θ, the tubes of generalized confining flux can be made thin which gives rise to a higher degree of confinement of any generalized color flux by dynamically condensed vacuum.

4 Behaviour of Dyons Around RCD Strings

Dyonically condensed vacuum of action (3.9) in characterized by the presence of two massive modes. The mass of scalar mode, M_ϕ given by (3.5), determines how fast the perturbative vacuum around a colored source reaches condensation and the mass M_D of vector mode determines the penetration length of the colored flux. The masses of these generalized dyonic glue balls may be estimated [18, 21, 35] by evaluated string tension of the classical string solution of quark pairs, since the extended Abelian Higgs model, described by action (3.9) in restricted chromodynamics admits string like solutions [36]. Let us examine the behaviour of dyons around such RCD strings. The classical field equations (3.10) and (3.11) contain a solution corresponding to the RCD string with a quark and an anti-quark at it ends. We consider such strings which are stationary and translationally invariant along the third direction $z = x^3$ of the reference frame used in action (3.9). Let us consider following Ansatz [26, 27] for the four components of the vector field W_μ and the two complex components ϕ_1 and ϕ_2 of the Higgs field ϕ,

$$W_\mu = \{Wi(\rho), W_\alpha(\rho)\} \quad \text{and} \tag{4.1}$$

$$\phi_i = f_i(\rho)[e^{i(\omega\alpha x\alpha)\delta i2}]e^{i\Psi(\rho)} \tag{4.2}$$

where $i = 1, 2$, $\alpha = 3, 4$, $f_i(\rho)$ are complex functions of $\rho = (x_1^2 + x_2^2)^{1/2}$, and ω_3 and ω_4 are real parameters. Here ω_4 is the relative rotation and ω_3 is the relative twist along z-axis between the components ϕ_1 and ϕ_2 of the Higgs field ϕ. This Ansatz breaks the originally present global $SU(2)$ symmetry to $U(1)$ and the various terms of the Lagrangian equation (3.8) reduces from four-dimensional configuration to the two-dimensional configuration in the following manner;

$$\begin{aligned} H_{\mu\upsilon}H^{\mu\upsilon} &\to H_{ij}^{2-}2[(\partial_i W_\alpha)(\partial_i W^\alpha)], \\ (D_\mu\phi)(D^\mu\phi) &\to \omega^\alpha(\omega_{\alpha+}2W_\alpha)|\phi_2|^2 - |D_i\phi_a|^2 + W_\alpha W^\alpha|\phi_a|^2 \quad \text{and} \\ [|\phi|^2 - 1]^2 &\to [|\phi_a|^2 - 1]^2 \end{aligned} \tag{4.3}$$

where

$$i, j \text{ and } a = 1, 2, \qquad W_\alpha W^\alpha = W_3^2 - W_4^2.$$

Then the action (3.9) reduces to

$$A \to v^2/|q|^2 \int dx^4 dx^3 \int d^2x \Big[W_\alpha W^\alpha |\phi_2|^2 + \frac{1}{2}\omega^\alpha(\omega_\alpha + 2W_\alpha)|\phi_2|^2 - \frac{1}{2}|D_i\phi_a|^2 + \frac{1}{4}H_{ij}^2 - \frac{1}{2}(\partial_i W_\alpha)(\partial_i W^\alpha) + \beta(|\phi_a|^2 - 1)^2 \Big] \tag{4.4}$$

where $H_{12} = -H_{21} = \partial W_2/\partial_{x1} - \partial W_1/\partial x_2$.

With this Ansatz the field equations (3.10) and (3.11) take the following forms in the $x_1 - x_2$ plane

$$\Box W_\alpha = -\omega_\alpha|\phi_2|^2 - W_\alpha|\phi_a|^2, \tag{4.5}$$

$$\partial_j H_{jk} = i[\phi_a D_k \phi_a], \tag{4.6}$$

$$D_i^2\phi_1 = -4\beta[|\phi_a|^2 - 1]\phi_1 - 2W_\alpha W^\alpha \phi_1 \quad \text{and} \tag{4.7}$$

$$D_2^2\phi_2 = -4\beta[|\phi_a|^2 - 1]\phi_2 - 2W_\alpha W^\alpha \phi_2 + \omega^\alpha(\omega_\alpha + 2W_\alpha)\phi_2, \tag{4.8}$$

where (4.5) may also be written as

$$\triangle W_\alpha = \omega_\alpha|\phi_2|^2 + W_\alpha|\phi_\alpha|^2 \tag{4.9}$$

with $\triangle = -\Box$.

Let us consider the solutions of these equations in the following simple cases of the Ansatz used in (4.1) and (4.2);

$$\text{(a)} \quad W_1 = \hat{x}_2 h(\rho)/|q|\rho^2; \qquad W_2 = -\hat{x}_1 h(\rho)/|q|\rho^2; \qquad W_3 = 0, \qquad W_4 = 0 \tag{4.10}$$

where $\hat{x}_1$ and $\hat{x}_2$ are unit vectors along $\hat{x}_1$ and $\hat{x}_2$ directions.

In this case (4.9) gives

$$\omega_\alpha = 0 \tag{4.11}$$

and then (4.7) and (4.8) reduce to

$$D_i^2\phi_a = -4\beta[|\phi_a|^2 - 1]\phi_a \tag{4.12}$$

and relation (4.2) becomes

$$\phi_i = f_i(\rho)e^{i\Psi(\rho)} \tag{4.13}$$

showing that there is neither relative rotation nor relative twist between the components ϕ_1 and ϕ_2 of the Higgs field ϕ. The solution (4.10) and (4.13) are static and untwisted semilocal solutions.

$$\text{(b)} \quad W_3 \neq 0, \qquad W_4 \neq 0 \quad \text{but} \quad \omega_\alpha\omega^\alpha = 0 \quad \text{and} \quad W_\alpha W^\alpha = 0. \tag{4.14}$$

In this case equation (4.9), when multiplied by ω^α and summed over α, becomes

$$\triangle(\omega^\alpha W_\alpha) = (\omega^\alpha W_\alpha)|\phi_a|^2$$

which has the trivial solution

$$\begin{aligned} &\omega^\alpha W_\alpha = 0 \quad \text{or} \\ &\omega^3 W_3 - \omega^4 W_4 = 0. \end{aligned} \tag{4.15}$$

Under these conditions equations (4.7) and (4.8) reduce to (4.12) for untwisted and static semilocal solutions.

$$\text{(c)} \quad W_3 \neq 0, \qquad W_4 \neq 0 \quad \text{but} \quad \triangle W_\alpha = 0. \tag{4.16}$$

Under these conditions a relation (4.9) gives

$$\omega_\alpha = -W_\alpha |\phi_a|^2/|\phi_a|^2 = -W_\alpha \frac{[|\phi_1|^2 + |\phi_2|^2]}{|\phi_2|^2}$$

or

$$\omega_\alpha = \frac{-W_\alpha v^2}{|\phi_2|^2} = \frac{-W_\alpha v^2}{|f_2|^2} \tag{4.16a}$$

showing that W_4 is larger for more relative phase between ϕ_1 and ϕ_2 and ϕ_3 is larger for more twist along z-axis. In this case solutions (4.5), (4.6), (4.7) and (4.8) are not static but stationary and twisted solutions.

Let us now examine the behaviour of dyons around the RCD strings with a quark and an anti-quark at its ends. It correspond to the untwisted and static solutions for the case given by (4.10) and (4.13), where ρ is the transverse distance to the string and

$$\Psi = \arg(x_1 + i x_2); \tag{4.17}$$

$$\begin{aligned} &\lim_{\rho\to 0} f(\rho) = \lim_{\rho\to 0} h(\rho) = 0; \\ &\lim_{\rho\to \infty} f(\rho) = \lim_{\rho\to \infty} h(\rho) = 1 \end{aligned} \tag{4.18}$$

where

$$f(\rho) = f_1(\rho) = f_2(\rho). \tag{4.19}$$

From (3.10), we have the magnetic constituent of the dyonic current as

$$k_\mu = |q| \mathrm{Im}[\phi^+ D_\mu \phi] = |q||\phi|^2[\partial_\mu \arg\phi + |q| W_\mu]. \tag{4.20}$$

Equation (4.17) gives

$$\partial\Psi/\partial x_2 = -x_1/\rho^2 \quad \text{and} \quad \partial\Psi/\partial x_2 = x_2/\rho^2. \tag{4.21}$$

Substituting relations (4.13), (4.10), (4.17), (4.19), and (4.21), into (4.20), we get

$$\begin{aligned} &k_1 = -(v^2 x_2/\rho^2)|q| f^2(\rho)[1 - h(\rho)]; \\ &k_2 = (v^2 x_1/\rho^2)|q| f^2(\rho)[1 - h(\rho)]; \\ &k_3 = 0; \qquad k_4 = 0, \end{aligned} \tag{4.22}$$

which gives the components of magnetic constituent of the dyonic current of the static untwisted solutions for RCD strings.

Substituting relations (4.13), (4.10) and (4.21) into field equation (4.12), we have

$$f''(\rho) + f'(\rho)/\rho - f(\rho)/\rho^2[1 - h(\rho]^2 + (M_\phi^2/2)[1 - f^2(\rho)] f(\rho) = 0, \tag{4.23}$$

where dash devotes derivatives with respect to ρ. At large distance, in view of (4.18), we may have

$$f(\rho) = 1 - \varepsilon(\rho), \tag{4.24}$$

where $\varepsilon(\rho)$ is infinitesimally small at large distance such that

$$\lim_{\rho\to\infty} \varepsilon(\rho) = 0.$$

Then (4.23) may be written as

$$\varepsilon''(\rho) + \varepsilon'(\rho)/\rho - M_\phi^2 \varepsilon(\rho) = 0.$$

Substituting $r = M_\phi \rho$ into this equation, we get

$$d^2\varepsilon(r)/dr^2 + \left(\frac{1}{r}\right) d\varepsilon(r)/dr - \varepsilon(r) = 0$$

which is modified Bessel's equation of zero order, with its solution given as

$$\varepsilon(r) = A I_0(r) = A I_0(M_\phi \rho), \tag{4.25}$$

where $I_0(r)$ is the modified Bessel's function of zero order, defined as

$$I_0(m_\phi \rho) = \sum_{n=0}^{\infty} \frac{(M_\phi \rho/2)^{2n}}{(n!)^2} = J_0(i M_\phi \rho), \tag{4.26}$$

with $J_0(x)$ as the ordinary Bessel's function of zero order.

In the similar manner, the field equation (3.12) may be written into the following form by using relations (4.10) and (4.22);

$$h''(\rho) - h'(\rho)/\rho + M_D^2[1 - h(\rho)] f^2(\rho) = 0. \tag{4.27}$$

At large distance we may have

$$h(\rho) = 1 - \zeta(\rho), \tag{4.28}$$

where $\lim_{\rho\to\infty} \zeta(\rho) = 0$.

Then (4.27) reduces to

$$\frac{d^2\zeta(r)}{dr^2} - \frac{d\zeta(r)}{dr} - \zeta(r) = 0, \tag{4.29}$$

where $r = M_D \rho$. Let us substitute $\zeta(r) = r\chi(r)$ is to this equation. Then we have

$$r d^2\chi(r)/dr^2 + d\chi(r)/dr - \chi(r)[1 + 1/r^2] = 0 \tag{4.30}$$

which is modified Bessel's equation of order-one with its solution given by

$$\chi(r) = \zeta(r)/r = B I_1(r) \tag{4.31}$$

where $I_1(r)$ is modified Bessel's function of order one defined as

$$I_1(r) = \frac{1}{i} J_1(ir) = \sum_{n=0}^{\infty} \frac{(r/2)^{2n+1}}{n!(n+1)!} \tag{4.32}$$

with $J_1(x)$ as the ordinary Bessel's function of first kind of order-one.

Thus we have

$$\zeta(\rho) = B(M_D\rho) I_1(M_D\rho) \tag{4.33}$$

with relations (4.25) and (4.33) the mistakes in the similar relations of Chernodub et al. [28] stand corrected.

Substituting relations (4.25) and (4.33) into (4.24) and (4.28), we have, at large value of ρ,

$$\begin{aligned} f(\rho) &= 1 - A I_0(M_\phi \rho) \quad \text{and} \\ h(\rho) &= 1 - B(M_D\rho) I_1(M_D\rho). \end{aligned} \tag{4.34}$$

Substituting these results into (4.10) and (4.13), with (4.19), we get the solution of classical field equation (3.12) and (4.12) corresponding to the RCD string with a quark and an anti-quark at its ends. The infinitely separated quark and anti-quark correspond to an axially symmetric solution of the string. For such a string solution with a lowest non-trivial flux the coefficient A in the solution (4.25) is always equal to one while the coefficient B in the solution (4.33) is unity in the Bogomolnyi limit exactly on the border between the type I and type II superconductors [37] where $M_D = M_\phi$ i.e. coherence length and the penetration length coincide with each other. Thus in RCD close to border, we set $B = 1$ besides $A = 1$ and then we have

$$f(\rho) = 1 - I_0(M_\phi \rho) = -\sum_{n=1}^{\infty} \frac{(M_\phi \rho/2)^{2n}}{(n!)^2}$$

and

$$h(\rho) = 1 - (M_D\rho) I_1(M_D\rho) = 1 - \frac{(M_D\rho/2)^2}{2} - M_D\rho \sum_{n=1}^{\infty} \frac{(M_D\rho/2)^{2n+1}}{n!(n+1)!}. \tag{4.35}$$

The RCD string is well defined by these solutions. In view of conditions (4.18), the magnetic constituent of the dyonic current, given by (4.22) near the RCD string, is zero at the centre of the string (i.e. for $x_1 = x_2 = 0$) and also zero at the points far away from the string (where $h(\rho) \to 1$). This current has a maximum at that transverse distance from the string for which the following conditions are satisfied;

$$\begin{aligned} &2(1-h) f f' - f^2 h' = 0 \quad \text{and} \\ &f^2 h'' - 2 f'(1-h) > \rho^2 f f''(1-h). \end{aligned} \tag{4.36}$$

5 Discussion

Equations (2.8) and (2.9) show that in the magnetic gauge the topological properties of $\hat{m}$ can be brought down to the dynamical variable W_μ by removing all non-essential gauge degrees of freedom in restricted chromo-dynamics (RCD) and hence the topological structure of the theory may be brought into dynamics explicitly. It assures a non-trivial dual structure of the theory of dyons in the magnetic gauge where the gauge fields are expressible in terms of purely time-like non-singular physical potentials V_μ and W_μ. Thus the topological charge in this theory may be identified as dual object of usual Noetherian charge. Consequently, the restricted theory is expected to lead to better insight of the complicated non-Abelian theory of dyons. The Lagrangian, given by (2.10) for RCD in magnetic gauge in the absence of quarks or any colored objects, establishes an analogy between super-conductivity and the dynamical breaking of magnetic symmetry which incorporates the confinement phase in RCD vacuum where the effective potential $V(\theta^*\theta)$ given by (3.1) induces the dyonic condensation of vacuum. This gives rise to dyonic super-current. The electic constituent of this current (i.e. its real part) screens the electric flux and confines the magnetic charges due to usual Meissner effect while its imaginary part (i.e. its magnetic constituent) screens the magnetic flux and confines the color iso-charges as the result of dual Meissner effect. Thus the dynamical breaking of the magnetic symmetry in this theory ultimately induces the generalized Meissner effect with electric constituent as the usual Meissner effect and its magnetic constituent as the dual Meissner effect. It dictates the mechanism for the confinement of the electric and magnetic fluxes associated with dyonic quarks [38] in the present theory. The confinement of colour is due to the spontaneous breaking of magnetic symmetry which yields a non-vanishing magnetically charged Higgs condensate where the broken magnetic group is chosen by abelianmization process demonstrated by (2.7), (2.8) and (2.9). It shows that the dyonic condensation mechanism of confinement in RCD is dominated by Abelian degrees of freedom, such Abelian dominance in connection with monopole condensation has recently been demonstrated by Boykov et al. [39]. The similar result has also been obtained more recently in a dual superconductivity model [32] which places the Y–M vacuum close to the border between type-I and type-II superconductors and marginally on the type-II side. Lagrangian equation (2.10), with effective potential given (3.1) describes the extended Abelian Higgs model which belongs to the interesting class of models in which both the global $SU(2)$ and local $U(1)$ symmetry are broken simultaneously with the minimal number of scaler field [26].

In the confinement phase of RCD, the dyons are condensed under the condition (3.2) where the transition from $\langle\phi\rangle_0 = 0$ to $\langle\phi\rangle_0 = v \neq 0$ is of first order, which leads to the vacuum becoming a chromo-magnetic super-conductor in the analogy with Higgs-Ginsburg Landau theory of super-conductivity. Dyonically condensed vacuum is characterized by the presence of two massive modes given by (3.3) and (3.4) respectively, where the mass of scalar mode M_ϕ determines how fast the perturbative vacuum around a color source reaches condensation and the mass M_D of vector mode determines the penetration length of the colored flux. With these two mass scales of dual gauge boson and dyonic field, the coherence length ε and the penetration length λ have been constructed by (3.5) in RCD theory. These two lengths coincide at the border between type-I and type-II super-conductors. In general, the ratio of penetration length and coherence length distinguishes superconductors of type-I ($\lambda < 6$) from type-II ($\lambda > 6$). Equation (3.11) gives the flux penetration depth in the dyonic model of RCD and shows that due to the dynamical breaking of magnetic symmetry, the vacuum acquires the properties similar to those of relativistic super-conductor where the quantum fields generate non-zero expectation values and induces screening currents. This penetration length excludes the generalized field in a manner similar to that in type II super-conductor where the appropriate screening currents are set up by the formation of Cooper's pairs giving rise to Meissner effect of magnetic flux confinement. Thus the generalized color flux is squeezed into flux tubes as a result of generalized Meissner effect caused by the coherence plasma of dyons in RCD vacuum which ultimately forces the quark (color) confinement in RCD. The generation of screening current and the finite range force field responsible for the confinement here are similar to those in the case of real electromagnetic super-conductor (i.e. relativistic superconductor). This approach of determining penetration length from the Lagrangian equation (3.8) of the extended Abelian Higgs model is restricted chromodynamics is much more direct and straight forward than that of lattice analysis [33–35] of the flux tube formed between two static color charges.

Equations (3.16) and (3.17) show that with the suitable choice of the generalized charge space parameter θ, the tubes of generalized confining flux can be made thin which gives rise to a higher degree of confinement of any generalized color flux by dynamically condensed vacuum. These equations demonstrate that the generalized charges lying on the cone of vertical angle $\theta = \pi/4$ in charge space give rise to thin tubes of confined color flux leading to strong confinement of the colored sources in RCD vacuum. On the other hand the generalized charges lying outside such cone and still participating in the vacuum condensation, immediately after magnetic symmetry breaking, have weak confinement effects. The generalized charge space parameter θ associated with dyons has the remarkable ability to squeeze the color fluxes and to improve the confining properties of RCD vacuum. Thus a perfect confinement can be achieved with pure dyonic states participating in actual dyonic condensation of RCD vacuum as the result of magnetic symmetry breaking in strong coupling limit.

The Ansatz given by relations (4.1) and (4.2) shows that there is a non-trivial coordinate dependent relative phase between the components of $SU(2)$ doublet. This anastz breaks the originally present global $SU(2)$ symmetry to $U(1)$ and reduces the four-dimensional action (3.9) to the two dimensional one given by (4.4), with the field equations given by (4.5), (4.6), (4.7) and (4.8). For the special case with the static solutions given by (4.10), (4.12) and (4.13) there is neither a relative rotation nor a relative twist between the components of Higgs field. In other special case, given conditions (4.14), also we got untwisted and semilocal solutions. in another case with the conditions (4.16), we got the stationary and twisted solution with the relative phase and twist along z-direction between the components of Higgs field given by relations (4.16a) showing that the component W_4 of the vector potential in RCD theory is larger for more relative phase between ϕ_1 and ϕ_2 while W_3 component is larger for more twist along z-directions.

Relations (4.25) and (4.33) removes the mistakes of the similar relations of Chernodub et al. [28]. Substituting relations (4.34) into (4.10) and (4.13), the solutions of classical field equations (3.7) and (3.8), corresponding to the RCD string with a quark and antiquark at its ends, readily follows. The RCD string is well defined by solutions (4.35) where the magnetic constituent of the dyonic current, given by (4.20) near the RCD string, is zero at the centre of the string and also zero at points far away from the string. This current is maximum at the transverse distance for which the conditions (4.36) are satisfied. The numerical value of this distance has been found to be about .2 fm corresponding to $SU(2)$ gluon dynamics [28]. Dyonic density in the absence of string has the contributions from monopole condensate [43, 44] and also from the perturbative fluctuations. According to (4.22) and (4.35) the magnetic constituent of dyonic current at large transverse distance form the string should be controlled by the coherence length and the penetration length where the coherence length could be derived directly from the analysis of dyonic density around a chromo-dyonic flux spanned between a static quark- anti quark pair. The coherence could also be derived alternatively either through some global fit to the whole set of Ginzburg-Landau equation [45, 46] or through the analysis of the temporal correlator of an observable directly coupled to it [32]. In the maximal Abelian gauge, as used in RCD here, the penetration length and coherence length are almost the same and hence the vacuum is nearly the border between type I and type II dual superconductors. We expect that these solutions (4.35) and the conditions (4.36) are stable in RCD model, however a detailed stability analysis in necessary to make definite conclusions. This problem will be undertake in our forthcoming paper.

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